Determinants
Part - 2

ASSERTION AND REASON BASED MCQs (1 Mark each)

Directions: In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false and R is True

Q. 1. Let A be a 2 × 2 matrix.

Assertion (A): $adj\ (adj\ A) = A$

Reason (R): $|adj\ A| = |A|$

Ans. Option (B) is correct.

Explanation:

$$adj\ (adj\ A) = |A|^{n-2} A$$

Here $n = 2 \Rightarrow adj\ (adj\ A) = A$

Hence A is true.

$$|adj\ A| = |A|^{n-1}$$

$$n = 2 \Rightarrow |adj\ A| = |A|$$

Hence R is true.

R is not the correct explanation for A.

Q. 2. Assertion (A): If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$, then

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}.$$

Reason (R): The inverse of an invertible diagonal matrix is a diagonal matrix.

Ans. Option (B) is correct.

Explanation:

$$|A| = 24$$

$$Adj\ A = \begin{bmatrix} 12 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(adj\,A) = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$$

Hence A is true.

A is a diagonal matrix and its inverse is also a diagonal matrix. Hence R is true.
But R is not the correct explanation of A.

Q. 3. Assertion (A): If every element of a third order determinant of value Δ is multiplied by 5, then the value of the new determinant is 125Δ.

Reason (R): If k is a scalar and A is an $n \times n$ matrix, then $|kA| = k^n |A|$

Ans. Option (A) is correct.

Explanation: If k is a scalar and A is an $n \times n$ matrix, then $|kA| = k^n |A|$.
This is a property of the determinant. Hence R is true.
Using this property, $|5\Delta| = 5^3 \Delta = 125\Delta$
Hence A is true.
R is the correct explanation of A.

Q. 4. Assertion (A): If the matrix $A = \begin{bmatrix} 1 & 3 & \lambda+2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{bmatrix}$ is singular, then $\lambda = 4$.

Reason (R): If A is a singular matrix, then $|A| = 0$.

Ans. Option (A) is correct.

Explanation: A matrix is said to be singular if

$$|A| = 0.$$

Hence R is true.

$$\begin{vmatrix} 1 & 3 & \lambda+2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{vmatrix} = 0$$

$$\Rightarrow \quad 1(40-40) - 3(20-24) + (\lambda+2)(10-12) = 0$$

$$0 + 12 - 2\lambda - 4 = 0$$

$$\Rightarrow \quad \lambda = 4.$$

Hence A is true.
R is the correct explanation for A.

Q. 5. Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$.

Assertion (A): $2A^{-1} = 9I - A$

Reason (R): $A^{-1} = \frac{1}{|A|}(adj\,A)$

Ans. Option (A) is correct.

Explanation: $A^{-1} = \frac{1}{|A|}(adj\,A)$ is true.

Hence R is true

$$|A| = 2,$$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

$$\text{LHS} = 2A^{-1} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix},$$

$$\text{RHS} = 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

$\therefore \quad 2A^{-1} = 9I - A$ is true.

R is the correct explanation for A.

Q. 6. Assertion (A): If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ and $A^{-1} = kA$, then $k = \frac{1}{9}$

Reason (R): $|A^{-1}| = \frac{1}{|A|}$

Ans. Option (D) is correct.

Explanation:

$$|A| = -4 - 15$$

$$= -19$$

$$A^{-1} = \frac{-1}{19}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$$

$$\Rightarrow \quad \frac{-1}{19}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} 2k & 3k \\ 5k & -2k \end{bmatrix}$$

$$\Rightarrow \quad k = \frac{1}{19}$$

A is false

$|A^{-1}| = \frac{1}{|A|}$ is true.

R is true.